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Why Do We Teach Calculus?

David M. Bressoud

The chimera of a course in discrete mathematics to replace freshman calculus raised its head briefly in the early 1980s and drew forth the defenders of calculus. Ronald Douglas, Daniel Kleitman, Peter Lax, Saunders MacLane, and others [1] have eloquently defended the necessity of placing calculus at the heart of the college mathematics curriculum. The issue seems settled, witness the Committee on the Undergraduate Program in Mathematics (CUPM) report reprinted in Reshaping College Mathematics [2] which affirms their position. I agree, but we are not done. If we are to accomplish the systemic changes that are needed in undergraduate education, then we must be clear about why we teach calculus.

The CUPM recommendation “to make no substantive changes in the first semester of calculus” is wrong. This course is not adequate as it stands. Our students approach calculus with a mixture of trepidation and anticipation. They know that it is going to be hard, but they also expect that this will be the course that draws together the mathematics that they have learned and transforms it into an instrument for comprehending the world around us. We know that this tool exists, but our students usually miss it. They leave disillusioned and disappointed. This past year I taught Advanced Placement AB (first semester) Calculus at our local high school. It gave me time to reflect on and experiment with my own response to the question in the title. I have two answers.

The first is that calculus is used in a variety of contexts by many disciplines. If we mathematicians did not teach it, others would have to. That is the essence of Lax’s article and the thrust of Douglas’s. It is an answer that is widely given and is being acted upon. Physicists, engineers, and biologists are being brought into our discussion of calculus reform. Textbooks are using real applications, and there is now rich source material [3]. Our use of this material is often faulty—too frequently it is tacked on rather than incorporated into the motivation for the concept it is to convey—but there is effort and progress in reforming calculus in this direction.

But, the usefulness of calculus is not a sufficient answer to my question. There are topics from discrete mathematics—statistical analysis, linear programming—that are far more useful to most of our students. My second answer, the one that has radical consequences for the way we teach calculus, is that calculus lies at the foundation of our scientific world view. Modern scientific thought has been formed from the concepts of calculus and is meaningless outside this context. When I speak of science, I do not restrict myself to other disciplines. In a very significant respect, mathematics itself came into being with the development of calculus.

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Sitting at the core of any modern education, mathematicians gaze back to ancient Babylon, Egypt, and Greece and preen themselves, secure in the delusion of an exalted position that has endured through the ages. In fact, there was no chair of mathematics at Oxford until 1619, nor at Cambridge until 1662. To the gentry of the mid-seventeenth century, the advantage was to Cambridge. Anthony à Wood describes this period: “Here by the way it must be remembered that the generality of the people some years before did verily think that the most useful branches of mathematics were spells and her professors limbs of the devil [4].” Samuel Pepys graduated from Cambridge ignorant of the multiplication table [5]. John Wallis would write of mathematics in the 1630s and 1640s at Cambridge: “[They were] scarce looked upon as Academical studies, but rather Mechanical; as the business of Traders, Merchants, Seaman, Carpenters, Surveyors of Lands, or the like, and perhaps some Almanack Makers in London [6].”

What changed this attitude was Newton’s Philosophiae Naturalis Principia Mathematica. It captured the public imagination in its revelation, explanation, and prediction of the phenomena of celestial mechanics. Suddenly, mathematics was being applied to the secrets of nature wherever they lay. One is struck by the exuberance of eighteenth century mathematics. We teach calculus because it is important for an understanding of who we are as a society.

We do a tremendous disservice to our students in the first year of calculus if we do not convey this excitement. I began my high school class with a discussion of why Principia is so important and concluded it with the proof that Kepler’s laws imply the law of gravity [7], a simple and elegant illustration of the power that arises from recognizing acceleration as the second derivative of position. I brought in simple differential equations at every opportunity and tried to introduce each new concept with its original purpose: Fermat was led to discover the derivative not because it gave him the slope of the tangent but because it identified local extrema; integration in the 1700s was about antidifferentiation, not finding areas and volumes.

History also tells me what I should not teach or, at the least, what I should approach with great caution: anything that follows Joseph Fourier’s Theory of the Propagation of Heat in Solid Bodies of 1807. Euler, Lagrange, and Cauchy committed great errors in their ignorance of the analysis that was developed in the nineteenth century, but the first year of calculus is not the time to describe these potential pitfalls. I would rather a student share Euler’s flare for manipulating series than memorize convergence tests. If we draw the line at 1807, then we do not need careful definitions of function, limit, and continuity. We can postpone the intermediate value theorem and satisfy our students with a heuristic understanding of the mean value theorem. I am willing to go over the line to admit the definite integral, introduced by Fourier in 1816, but a description of the Riemann integral is out of bounds.

A historical pedagogy should not be applied with rigidity. Differential forms make sense of vector calculus, but we cannot begin the study of vector calculus with differential forms and neither should we forget the effort required to achieve the modern sense of rigor in calculus or ignore the reasons that made it necessary. Here, I follow Henri Poincaré:

The task of the educator is to make the child’s spirit travel again where his fathers have passed, crossing certain stages rapidly but suppressing none of them. In this regard, the history of science must be our guide [8].
REFERENCES


6. John Wallis, *An Account of Some Passages in His Own Life*, 1697, as quoted in Howson, *ibid*.


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Whoops!

In the April '92 issue of this *MONTHLY*, we announced Slowinski's discovery of the most recent Mersenne prime \((2^{756839} - 1)\) and declared it to have 227,831 digits. Several people have written in to point out the number is in fact larger than this. Attentive reader Charles Vanden Eynden was the first. He wrote a (polite) letter pointing out that every student in his elementary number theory class quickly calculated the number of digits as 227,832 since they realized the number of digits was one greater than the log base 10 of the number. The *MONTHLY* apologizes for the error.