Ch. 6 In-Class Exercise

According to Coulomb’s Law, two charged particles repel or attract by an amount which is related to the potential energy \( V \) between them.

\[
V = k \frac{q_1 q_2}{d}
\]

where \( q_1 \) and \( q_2 \) are the charges on the particles, \( d \) is the distance separating the particles and \( k \) is just a positive-valued proportionality constant.

Figure 1. Two charged particles separated by a distance “d”.

If \( q_1 \) and \( q_2 \) have opposite signs (one positive and the other negative), the potential energy will be negative and therefore attractive.

1. Assuming that \( q_1 \) and \( q_2 \) remain constant, what happens to the magnitude of \( V \) if the separation, \( d \), is increased?
Let’s consider the case of an atom, where the **positive** particle is the **nucleus** and the **negative** particles is the **electron**.

2. a. If $q$ for an electron is $-1$, what is $q$ for a **proton**? ______

   b. What is $q$ for a **neutron**? ______

   c. What is $q$ for the nucleus of a **C** atom? ______

3. Recall that a $^1\text{H}$ atom consists of a **proton** as the nucleus and an **electron** outside of the nucleus. Is the **potential energy** of a hydrogen atom **positive** (repulsive) or **negative** (attractive)?

The **ionization energy** (IE) is the amount of energy needed to remove an electron from an atom. The following table shows the ionization energies for hydrogen atoms where the electron is at progressively larger distances from the nucleus.

<table>
<thead>
<tr>
<th>d (pm)</th>
<th>IE (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>$0.0462 \times 10^{-18}$</td>
</tr>
<tr>
<td>1000</td>
<td>$0.231 \times 10^{-18}$</td>
</tr>
<tr>
<td>500.0</td>
<td>$0.462 \times 10^{-18}$</td>
</tr>
<tr>
<td>200.0</td>
<td>$1.16 \times 10^{-18}$</td>
</tr>
<tr>
<td>100.0</td>
<td>$2.31 \times 10^{-18}$</td>
</tr>
</tbody>
</table>

4. Do you expect the **potential energy**, $V$, of these hydrogen atoms to be **positive** or **negative**? Explain.

5. Do you expect the **potential energy** of these hydrogen atoms to become **smaller** or **larger** as $d$ increases?

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$^1$ 1 pm = $10^{-12}$ m
6. Which of the following systems will have the larger ionization energy?
   a. an electron at a distance of 500 pm from a nucleus with charge +2.
   b. an electron at a distance of 700 pm from a nucleus with charge +2.

7. Which of the following systems will have the larger ionization energy?
   a. an electron at a distance of 500 pm from a nucleus with charge +1.
   b. an electron at a distance of 500 pm from a nucleus with charge +2.

8. Consider an H atom and a He\textsuperscript{+} ion. Which of these will have the larger ionization energy? Explain your reasoning.

For atoms with more than one electron around the nucleus, different ionization energies are required to remove different electrons. Of these, the smallest one is referred to as the first ionization energy, \( IE_1 \). This is the smallest energy required to remove an electron from an atom.

\[
M(g) \rightarrow M^+(g) + e^- 
\]

For a hydrogen atom, the first ionization energy is \( 2.178 \times 10^{-18} \) J.

\[
H(g) \rightarrow H^+(g) + e^- \quad IE_1 = 2.178 \times 10^{-18} \text{ J}
\]

9. Refer to Table 1: what is the distance between the nucleus and the electron in an H atom?
10. How much total energy is required to remove the electrons from a mole of H atoms? Show work.

11. Predict the relationship between IE$_1$ and atomic number by making a rough graph of IE$_1$ vs. atomic number.
Based on our previous examination of ionization energies, we would expect that the ionization energy of an atom would **increase** as the nuclear charge, $Z$, increases. In addition, the ionization energy of an atom should **decrease** if the electron being removed is moved farther away from the nucleus (that is, if $d$ increases).

Table 2 lists the experimentally measured ionization energies of the first 20 elements.

**Table 2. Ionization energies of the first 20 elements.**

<table>
<thead>
<tr>
<th>Z</th>
<th>IE$_1$ (MJ/mol)$^2$</th>
<th>Z</th>
<th>IE$_1$ (MJ/mol)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>H</td>
<td>11</td>
<td>Na</td>
</tr>
<tr>
<td>2</td>
<td>He</td>
<td>12</td>
<td>Mg</td>
</tr>
<tr>
<td>3</td>
<td>Li</td>
<td>13</td>
<td>Al</td>
</tr>
<tr>
<td>4</td>
<td>Be</td>
<td>14</td>
<td>Si</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>15</td>
<td>P</td>
</tr>
<tr>
<td>6</td>
<td>C</td>
<td>16</td>
<td>S</td>
</tr>
<tr>
<td>7</td>
<td>N</td>
<td>17</td>
<td>Cl</td>
</tr>
<tr>
<td>8</td>
<td>O</td>
<td>18</td>
<td>Ar</td>
</tr>
<tr>
<td>9</td>
<td>F</td>
<td>19</td>
<td>K</td>
</tr>
<tr>
<td>10</td>
<td>Ne</td>
<td>20</td>
<td>Ca</td>
</tr>
</tbody>
</table>

Figure 2 is a representation of the hydrogen atom according to the Bohr model, where a nucleus of charge +1 is attracted to an electron of charge -1 at a distance of 500 pm.

\[1\text{ MJ} = 10^6\text{ J}\]
As we have shown previously, the ionization energy of this atom is 1.31 MJ/mol. Examining the ionization energy of He (Z = 2), we note that its ionization energy is larger than that of H, by approximately a factor of 2. This is consistent with the two electrons in the He atom orbiting the He nucleus at a distance approximately the same as that in H, as shown in Figure 3.

For Li, there is a change in the trend of the ionization energy. The ionization energy of a Li atom is less than that of He. In fact, it is significantly smaller than that of the H atom! This is not consistent with a model of placing a third electron at about the same distance as the other electrons, for this would result in an ionization energy which is larger than that of He.
In order for Li to have a lower first ionization energy than H, either the nuclear charge, \(Z\), must be lower than that of H, or the distance of the easiest-to-remove electron from the nucleus must be greater than in H (and He). We know that the nuclear charge is not lower than that of H; thus, the electron being removed from the Li atom must be farther from the nucleus than in H or He. This model is shown in Figure 4.

**Figure 4. Schematic diagram of a lithium atom.**

Notice that the outermost electron (which is the easiest-to-remove electron) is farther from the nucleus than the two inner electrons. Since all electrons repel each other because they are negatively charged, the outer electron is repelled by the two inner electrons. This dramatically decreases the overall force of attraction pulling the outer electron toward the nucleus. Thus, the net charge (or core charge) acting on the outer electron (to hold onto it) is the charge of the nucleus (+3) minus the charges of the inner electrons (-1 each).

\[
\text{net charge or core charge on outermost electron} = (+3) + (2 \times -1) = +1
\]
The electrons in the outermost region of the atom are called **valence** electrons; the other electrons, which are closer to the nucleus, are called **core** electrons. Li has two core electrons and one valence electron.

\[
\text{CHARGE ACTING ON VALENCE ELECTRONS} = \text{CORE CHARGE} = \\
\text{CHARGE ON NUCLEUS} + (\text{NUMBER OF CORE ELECTRONS} \times -1)
\]

The next element, Be, has an ionization energy which is larger than that for Li. This is consistent with the fourth electron in Be being at about the same distance from the nucleus as the third electron.

**Figure 5. Schematic diagram of a beryllium atom.**

![Schematic diagram of a beryllium atom](image)

The core charge on the beryllium valence electrons can be calculated as follows:

\[
\text{core charge} = +4 + (2 \times -1) = +2
\]

Although there are some slight variations, in general there is an increase in ionization energy as the atomic number increases up to \(Z = 10\) (Ne). This is consistent with an increase in core charge. Between \(Z = 4\) and \(Z = 10\) there is no large drop in ionization energy; this suggests that, as electrons are added to the atom, they exist at about the same distance from the nucleus.
In moving to atomic number 11, Na, we observe a dramatic drop in ionization energy from that of Ne (see Table 2). This decrease suggests that the eleventh electron in Na is farther away from the nucleus than any of the other electrons.
12. Calculate the core charge for
   a. Ne

   b. Na